

# Common English errors in mathematical papers

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## Articles

WRONG: *The function  $-e^{-x}$  ~~is derivative~~ of  $e^{-x}$ . The function  $-e^{-x}$  is ~~not~~ derivative of  $e^{-x}$ .*

RIGHT: *The function  $-e^{-x}$  is the derivative of  $e^{-x}$ .*

COMMENTS: The noun *derivative* is “countable”—it cannot occur without an article. The sentence with the indefinite article means that  $-e^{-x}$  is SOME derivative of the function  $e^{-x}$ , which does not make sense, because the derivative is uniquely determined.

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WRONG: *Let  $U'$  be ~~the~~ linear complement of the subspace  $U$  in  $V$ .*

RIGHT: *Let  $U'$  be a linear complement of the subspace  $U$  in  $V$ .*

COMMENTS: There are many complements of  $U$ ; if you have in mind any of them, you have to use the indefinite article. On the other hand, you can say: *Let  $U'$  be the linear complement of the subspace  $U$  in  $V$ , described in Remark 2*—here you are specifying WHICH complement you have in mind.

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WRONG: *~~Such operator~~ is defined by...*

RIGHT: *Such an operator is defined by...*

COMMENTS: The word *such*, when appearing before a singular countable noun, is followed by *a/an*.

EXCEPTIONS: This rule does not obey if *such* is preceded by a quantifier: *one such map; for every such map; some such difficulty*.

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WRONG: *In ~~the Section 2~~*

RIGHT: *In Section 2*

COMMENTS: If a series of objects are numbered by positive integers, corresponding to ordinal numbers, no article is used: *in Section 2; on page 4; in row  $n$* .

However, often the numbering/labelling is not as direct and then *the* may appear; e.g. usually we write *Definition 2.1*, but you can say both *inequality (2.1)* and *the inequality (2.1)*.

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WRONG: *~~The~~ closed sets are Borel sets.*

RIGHT: *Closed sets are Borel sets.*

COMMENTS: *The* does not mean “all”. If you talk about things in general, use no article.

EXCEPTIONS: This rule does not obey in some constructions with *of*—it is understood that the generality is somehow limited here:

- *The members of the collection  $U$  are called the open sets of  $X$ .*

Also, use *the* when you are talking about a set as a whole:

- *The linear operators on  $V$  can be identified with the matrix space  $M$ .*

WRONG: *The ~~number of the~~ solutions of (1); the ~~set of the~~ solutions of (1)*

RIGHT: *The number of solutions of (1); the set of solutions of (1)*

COMMENTS: On the other hand, you say e.g. *the union of the sets  $U_i$* .

## Singular or plural?

WRONG: *There is a ~~finite number~~ of elements such that...*

RIGHT: *There are a finite number of elements such that...*

COMMENTS: Here the quantifying expression *a finite number of* has the same meaning as *finitely many*, and it has the same syntax, i.e. it requires a plural verb.

## Which tense to use?

WRONG: *In 2008 Fox ~~has shown~~ that...*

RIGHT: *In 2008 Fox showed that...*

COMMENTS: If you are giving a date, it is understood that you are thinking about a definite moment in the past; you then have to use the *Simple Past* tense.

However, you can well say, without specifying the time: *Fox has shown that...*—Fox proved something in the past, but when talking about it, you are also thinking about the present: IT IS PROVED NOW, because he proved it (no matter when). In such circumstances, use the *Present Perfect* tense.

## Syntax of verbs

WRONG: *~~Let  $F$  denotes~~ a function such that...*

RIGHT: *Let  $F$  denote a function such that...*

COMMENTS: *Let* is the imperative of the verb *to let* and has to be followed by an infinitive (without *to*). You can also say: *We let  $F$  denote a function...* or *Let us denote by  $F$  a function...*

WRONG: *This lemma ~~allows to~~ prove the theorem without using (2).*

RIGHT: *This lemma allows us to prove the theorem without using (2).*

COMMENTS: The verb *allow* requires an indirect object: you have to say WHOM the lemma allows to prove the theorem. If you do not want to say that it allows you (“us”), you can say: *This lemma allows one to prove the theorem*, that is, it allows you and the reader.

You can avoid adding *us/one* by using a noun or an ing-form:

- *This lemma allows proving the theorem without the use of (2),*  
or the passive voice:
- *This lemma allows the theorem to be proved without using (2).*

The same problem concerns the verbs *enable* and *permit*. Here are examples of their correct use:

- *Repeated application of Lemma 2 enables us to write...*
- *Theorem 3 enables discontinuous derivations to be built.*
- *This will permit us to demonstrate that...*
- *Formula (6) permits transfer of the results in Section 2 to sums of i.i.d. variables.*

Another verb requiring an indirect object is *remind*:

WRONG: *The purpose of this section is to ~~remind some~~ results on...*

RIGHT: *The purpose of this section is to remind the reader of some results on...*

If you do not want to involve the reader, you can use *recall*:

- *The purpose of this section is to recall some results on...*

WRONG: *We should ~~avoid to use~~ (2) here, because...*

RIGHT: *We should avoid using (2) here, because...*

COMMENTS: After some verbs you cannot use an infinitive; they have to be followed by an ing-form. These include *avoid*, but also *finish* and *suggest*:

- *After having finished proving (2), we shall return to...*
- *This suggests investigating the solutions of...*

Here are other similar examples:

WRONG: *Section 3 is ~~devoted to prove~~ this theorem.*

RIGHT: *Section 3 is devoted to proving this theorem.*

WRONG: *The ~~possibility to obtain~~ a better bound*

RIGHT: *The possibility of obtaining a better bound*

## All that glitters is not gold

WRONG: *~~Every subspace of  $V$  is not of the form (3).~~*

RIGHT: *No subspace of  $V$  is of the form (3).*

COMMENTS: Using *all* or *every* with a negative statement is risky, as shown by the above proverb, which says of course that NOT ALL that glitters is gold, although at first sight it may seem (to a non-native speaker of English) that something is being stated about “all that glitters”. Therefore, constructions with *no*, *none*, *never* etc. are preferable.

## Every or any?

WRONG: *For ~~every two maps~~  $f$  and  $g$ ; for ~~every positive integers~~  $m$  and  $n$*

RIGHT: *For any two maps  $f$  and  $g$ ; for all positive integers  $m$  and  $n$*

COMMENTS: *Every* has to be followed by a singular noun: *for every map*.

EXCEPTIONS: The combination *every two* can appear when talking about frequency, e.g. *The government changes every two months*.

### **Not or non-?**

WRONG: A ~~*not empty set*~~

RIGHT: A *nonempty set*

COMMENTS: If you want to negate an adjective that appears before a noun (*attributive position*), you have to use *non-*: *a nonempty set*; *a non-locally convex space*; *a non-Euclidean domain*. Using *not* is only possible when the adjective follows the verb *be* (*predicative position*): *This space is not Euclidean*.

### **First or at first?**

WRONG: ~~*At first, we prove*~~ (2).

RIGHT: *First, we prove* (2).

COMMENTS: *At first* is used when you are talking about what happens in the early stages of an event, in contrast to what happens later: *It might seem at first that the noncompactness is not an obstacle*.

### **Use of prepositions**

WRONG: We can ~~*join a with b*~~ by a path  $\pi$ .

RIGHT: We can *join a to b* by a path  $\pi$ .

WRONG: ..., which ~~*contradicts to*~~ Theorem 2.

RIGHT: ..., which *contradicts* Theorem 2.

COMMENTS: The verb *contradict* is *transitive*: *to contradict something*, and not “to contradict to something” or “with something”. If you want to use *to* at the end of an indirect proof, you can write: ..., *contrary to* Theorem 2.

WRONG: ~~*Continuous in the point*~~  $x$

RIGHT: *Continuous at the point*  $x$

COMMENTS: But of course a function can be *continuous in the set*  $A$  (or *on the set*  $A$ ).

WRONG: ~~*Independent on*~~  $x$

RIGHT: *Independent of*  $x$

COMMENTS: On the other hand, we have: *depending on*  $x$ ; *independence of*  $F(U, V)$  *from*  $V$ .

WRONG: ~~*Disjoint with X*~~

RIGHT: *Disjoint from X*

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WRONG: ~~*Then F is equal B.*~~

RIGHT: *Then F is equal to B.*

*Then F equals B.*

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WRONG: ~~*We shall prove this in the end of Section 3.*~~

RIGHT: *We shall prove this at the end of Section 3.*

COMMENTS: *In the end* means finally, as a result of the previous situation, e.g. *Thus in the end, after all these transformations, F will be homogeneous.*

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WRONG: ~~*The coefficient by  $x^3$  in the expansion*~~

RIGHT: *The coefficient of  $x^3$  in the expansion*

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WRONG: ~~*Then F is greater or equal to 3.*~~

RIGHT: *Then F is greater than or equal to 3.*

COMMENTS: The adjectives *greater* and *equal* require different prepositions: “greater to 3” does not make sense. There are many ways of avoiding this clumsy construction: *F is at least/most 3*; *F does not exceed 3*; *F has no more/fewer than 3 elements*; *F is of degree 3 or less/more.*

## Walking in the street, the sun was shining

WRONG: *Setting  $x = 0$ , the assertion follows.*

RIGHT: *Setting  $x = 0$  yields the assertion.*

*Setting  $x = 0$ , we obtain the assertion.*

*If we set  $x = 0$ , the assertion follows.*

COMMENTS: The subject of a clause containing a participle (here, *Setting*) should be the same as the subject of the main clause (*the assertion does not set anything; we do*). This error is called an *unattached participle* or *dangling participle*.

EXCEPTIONS: “Unattached participles” are normal in some expressions referring to the speaker’s attitude, e.g.

- *Roughly speaking,...* *Considering the proof,...* *Assuming F is continuous,...*

## Word order

WRONG: ~~*Theorem 3 we shall prove in Section 4.*~~

RIGHT: *We shall prove Theorem 3 in Section 4.*

*Theorem 3 will be proved in Section 4.*

COMMENTS: The subject normally precedes the direct object.

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WRONG: *We can ~~prove easily~~ Theorem 3 by applying (2).*

*We will prove in ~~Section 4~~ Theorem 3.*

RIGHT: *We can easily prove Theorem 3 by applying (2).*

*We shall prove Theorem 3 in Section 4.*

COMMENTS: In general, it is best not to put anything between the *verb* and the *direct object*.

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WRONG: *A ~~bounded by 1~~ function*

RIGHT: *A function bounded by 1*

COMMENTS: If an expression qualifying a noun contains a preposition (here *by*), it has to follow the noun.

EXCEPTIONS: This rule is violated by certain expressions which are felt as one word, e.g. *a global in time solution*, previously written with hyphens: *a global-in-time solution*. Generally, any sequence of words joined with hyphens can play the role of an adjective, e.g. *the you-know-which map*; but you can hardly use this stylistic device systematically in a mathematical paper.

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WRONG: *The ~~two following~~ sets*

RIGHT: *The following two sets*

WRONG: *Let  $f$  be ~~such a function~~ that...*

RIGHT: *Let  $f$  be a function such that...*

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WRONG: *We now list all the ~~involved~~ functions.*

RIGHT: *We now list all the functions involved.*

COMMENTS: Past participles often appear after a noun, especially if they replace an identifying relative clause: *We now list all the functions that will be involved in our study.*

Other examples where the past participle must appear AFTER the noun: *the process described; the problem discussed/mentioned; the solution obtained/adopted.*

EXCEPTIONS: However, there are many past participles that can appear before nouns, e.g. *an involved explanation* = a complicated explanation (so the word *involved* changes its meaning according to its position); *the stated properties; the above-mentioned problem.*

## Wrong word used

WRONG: *This proves the ~~thesis of our~~ theorem.*

RIGHT: *This proves the assertion/conclusion of our theorem.*

COMMENTS: A *thesis* is either a dissertation, or an argument, theory etc. that can be accepted or rejected.

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WRONG: *~~To this aim~~, we first consider...*

RIGHT: *To this end, we first consider...*

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WRONG: We denote it ~~shortly~~ by  $A_f$ .

RIGHT: We denote it *briefly* by  $A_f$ .

COMMENTS: *Shortly* = soon. For example: *A precise definition will be given shortly.*

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WRONG: We ~~expose examples~~ of maps such that...

RIGHT: We *present/give examples* of maps such that...

COMMENTS: *Expose* = uncover, reveal; this word seldom appears in mathematical texts, as opposed to *exposition* = a comprehensive explanation (of a problem): *A detailed exposition is given in [5].*

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WRONG: The ~~function  $f$  verifies~~ equation (1).

RIGHT: The *function  $f$  satisfies* equation (1).

COMMENTS: In English, *a function cannot verify anything; only a person can.* Note that this construction is normal in French: *la fonction  $f$  vérifie l'équation (1).*

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WRONG: ~~Contrary to [7],~~ we do not assume the compactness of  $X$ .

RIGHT: *In contrast to [7],* we do not assume the compactness of  $X$ .

COMMENTS: The phrase *contrary to* is correctly used e.g. in indirect proofs: *Then  $f = 1$ , contrary to assumption.*

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WRONG: Then  $f = 1$ , ~~what completes~~ the proof.

RIGHT: Then  $f = 1$ , *which completes* the proof.

COMMENTS: On the other hand, you can say: *The function  $f$  is continuous; what is more, it is differentiable.* Where lies the difference between these two examples? *Which* refers to what precedes it in the sentence, while *what* refers to what follows it.

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WRONG: The function  $F$  ~~will be precised~~ in Section 2.

RIGHT: The function  $F$  *will be made precise* in Section 2.  
*The function  $F$  will be specified* in Section 2.

COMMENTS: It is tempting to translate the French *préciser* or Polish *sprecyzować* as “to precise”; unfortunately, there is no such word: *precise* is only an adjective.

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## Miscellanea

WRONG: In this paper we ~~prove among~~ others that...

RIGHT: In this paper we *prove among other things* that...

COMMENTS: *Among others* can only be used when it is clear what “others” you have in mind, e.g. *Our result generalizes Brown's theorem, among others.*

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WRONG: *There exists a limit  $\lim_{x \rightarrow 0} F(x)$ .*

RIGHT: *The limit  $\lim_{x \rightarrow 0} F(x)$  exists.*

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WRONG: *~~On the other side~~,  $F$  fails to have property  $P$ .*

RIGHT: *On the other hand,  $F$  fails to have property  $P$ .*

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WRONG: *~~As usually~~, we can rephrase this as a uniqueness theorem.*

RIGHT: *As usual, we can rephrase this as a uniqueness theorem.*

## Punctuation

WRONG: *We ~~prove, that~~...*

RIGHT: *We prove that...*

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WRONG: *Let  $f$  be any function, ~~which~~ satisfies condition (1).*

RIGHT: *Let  $f$  be any function which satisfies condition (1).*

COMMENTS: Do not put a comma before a “defining *which*”. On the other hand, put a comma before *which* if it starts a non-identifying clause; a typical example appears at the end of a proof: *Then  $f = 1$ , which completes the proof.*

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WRONG: *~~Let  $n \in \mathbb{N}$ , then~~...*

RIGHT: *Let  $n \in \mathbb{N}$ . Then...*

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WRONG: *This case has been thoroughly ~~studied, see~~ [2, 3, 8].*

RIGHT: *This case has been thoroughly studied (see [2, 3, 8]).*

*This case has been thoroughly studied; see [2, 3, 8] for more details and examples.*